

# Surface Plasmon Excitation on the Surface of RbFnano Tubes of Different Radius

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## ABSTRACT

Surface Plasmon waves depend on the nature and geometries of RbF materials. The behaviors of surface Plasmon are studied with help of dispersion relation for two modes by considering propagation constant using modified Hydro dynamical models. It is observed that surface Plasmon depend upon radius of RbF nanotubes and long wavelength of incident em waves on this tubes. For shorter wavelength, frequencies of surface Plasmon increases linearly which is independent on the radius of RbF nano tubes.

## 1. INTRODUCTION

Surface waves and coupling between them several workers have made attempts to explain the surface enhanced Raman Effect [1-2]. Super conductivity of high temp [28] fractional quantum Hall effect [3] and several other effects [4] has been study. These studies have very wide applications in integrated optics and microelectronics [5]. Therefore, it was thought of great interest to study of this surface Plasmon phonon coupling [17-24].

A survey of literature shows that generally scientists have derived the dispersion relation for curved geometries for the case  $\bar{K} = 0$  [6-9] ( $\bar{K}$  is the wave vector). The dispersion relation for SP-SOP modes of cylindrical polar semiconductor  $>0$  has been obtained as a particular case by using Bloch's hydrodynamic method [8-13].

## 2. DISPERSION RELATION FOR TWO MODE COUPLINGS

The hydrodynamic equations [7] provide a differential equation for the electron density fluctuation

' $n_l(\bar{r})$ ' in a bulk polar semiconductor which is given by

$$[\nabla^2 - \alpha^2]n_l(\bar{r}, t) = 0 \quad (1)$$

$$\text{Here } \alpha^2 = \frac{1}{\beta^2}(\omega_p^2 - \omega^2), \quad \omega_p = \left(\frac{4\pi n_0 e^2}{\epsilon m}\right)^{\frac{1}{2}} \quad \&$$

$\beta = V_F / \sqrt{3}$  are respectively the volume plasma frequency and the average speed of propagation of hydrodynamic disturbance in the electron gas.

The equation (1) is applicable for both planes as well as curved geometries and thus may be used to derive the dispersion relation for the coupled SP-SOP modes in a polar semiconducting material.

For a cylindrical polar semiconductor of radius 'R' the equilibrium electronic concentration  $n_l(\bar{r})$  must satisfy the following conditions -

$$\begin{aligned} n_l(r) &= n_0 \quad r < R \\ &= 0 \quad r > R \end{aligned}$$

The electron density fluctuation  $n_l(\bar{r})$  may be expanded in cylindrical polar coordinates as:

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$$n_l(\bar{r}) = \sum_{l,m} N_l(\bar{r}) Y_l^m(\theta, r) e^{i(\bar{k} \cdot \bar{r})} \quad (2)$$

Where  $N_l(\bar{r})$  is the radial part being an integer which determines the angular dependence,  $Y_l^m(\theta, r)$  is cylindrical harmonic and  $\bar{K}$  is the wave vector.

The equation (1) with the help of equation (2) gives the following two equations:

$$\frac{d^2 N_l(\bar{r})}{dr^2} + \frac{2}{r} \frac{dN_l(\bar{r})}{dr} - [p^2 + \frac{l(l+1)}{r^2}] N_l(\bar{r}) = 0 \quad (3)$$

$$\frac{d^2 N_l(\bar{r})}{dr^2} + \frac{1}{r} N_l(\bar{r}) = 0 \quad (4)$$

$$p^2 = \alpha^2 + K^2 \quad (5)$$

The equation (3) is the modified cylindrical Bessel's equation. With the help of this equation, the time dependent solution for  $n_l(\bar{r}, t)$ , the interior and exterior regions of the cylindrical, and may be written as

$$n_l(\bar{r}, t) = \sum_{lm} A_l I_l(\rho r) Y_l^m(\theta, r) e^{i(k \cdot r \omega_1)} \quad r < R$$

$$= 0 \quad r < R$$

The solution of equation (3) does not show the angular dependence i.e. the  $l$  dependence therefore; it does not lead to any physically important solution. The potential perturbation function  $\phi_l(\bar{r}, t)$  can be written in terms of electron density fluctuation  $n_l(\bar{r}, t)$  as

$$\nabla^2 \phi_l(\bar{r}, t) = \frac{4\pi e}{\bar{\epsilon}} n_l(\bar{r}, t) \quad (6)$$

The function  $\phi_l(\bar{r}, t)$  may also be expanded in polar co-ordinates as

$$\phi_l(\bar{r}, t) = \phi(r, \theta, t)$$

$$= \sum_{lm} \phi_l(\bar{r}, t) Y_l^m(\theta, r) e^{i(\bar{K} \cdot \bar{r} - \omega t)} \quad (7)$$

The equation (7) when substituted in equation (6), gives the solution for potential perturbation function  $\phi_l(\bar{r}, t)$  for the interior and exterior regions respectively. Now we apply the boundary conditions [24] as given

$$(i) \quad (\phi_{li})_{r=R} = \epsilon_2 \left( \frac{\partial}{\partial r} \phi_{le} \right)_{r=R}$$

$$(ii) \quad \epsilon_1 \left( \frac{\partial}{\partial r} \phi_{le} \right)_{r=R} = \epsilon_2 \left( \frac{\partial}{\partial r} \phi_{li} \right)_{r=R}$$

$$(iii) \quad \frac{e}{m} \frac{\partial}{\partial r} \phi_{li}(\bar{r}, t) = \frac{\beta^2}{n_0} \frac{\partial}{\partial r} n_l(\bar{r}, t) \quad \text{at } r = R \quad (8)$$

We obtain the dispersion relation for two modes coupling as

$$(\bar{\epsilon}A - \epsilon_2 B) \omega_p^2 + (\epsilon_2 C - \epsilon_1 A) \omega^2 = 0 \quad (9)$$

$$A = \frac{X_{l-1}(\rho) - X_{l+1}(\rho)}{lX_{l-1}(\rho) + (l+1)X_{l+1}(\rho)}$$

$$B = \frac{I_{l-1}(\rho) - I_{l+1}(\rho)}{lI_{l-1}(\rho) + (l+1)I_{l+1}(\rho)}$$

$$C = \frac{I_{l-1}(\rho) - I_{l+1}(\rho)}{lI_{l-1}(\rho) + (l+1)I_{l+1}(\rho)}$$

The integer 'l' signifies the mode of oscillation. Modes  $l = 1, 2, \dots$  correspond to dipole, quadrupole etc., oscillation respectively. The  $l = 0$  mode does not give any root for frequency  $\omega$  and thus the solutions start with  $l = 1$ .

### 3. DISPERSION RELATION FOR THE CASE $\bar{K} = 0$

The spatial dispersion relation for the case  $\bar{K} = 0$  can be obtained from equation (20) directly by substituting the value of constant A, B and C as  $\bar{K} \rightarrow 0$ , in the limit  $\bar{K} \rightarrow 0$  the value of constants A, B and C are given by

$$A = -\frac{1}{l+1}$$

$$B = \frac{I_{l-1}(\alpha R) - I_{l+1}(\alpha R)}{II_{l-1}(\alpha R) + (l+1)I_{l+1}(\alpha R)}$$

$$C = \frac{1}{l}$$

With the above substitutions the dispersion relation (9) reduces to

$$\frac{II_{l-1}(\alpha R) + (l+1)I_{(l+1)}(\alpha R)}{II_{l-1}(\alpha R)} = \frac{(2l+1)\epsilon_2\omega_p^2}{[\epsilon_L + (l+1)\epsilon_2]\omega^2 + [l(\epsilon_2 - \bar{\epsilon})]\omega_p^2} \quad (10)$$

Equation (10) is the dispersion relation for the coupled SP-SOP modes in a polar semi conducting sphere for the case  $\bar{K} = 0$ , ' $\epsilon_L$ ' is the background dielectric function of the medium and ' $\epsilon_2$ ' is the dielectric constant of the bounding medium. The dispersion relation (10) is the same which was derived by Barberan and Bausells [6] for metallic sphere.

We use the local form of dielectric function which can be written as

$$\epsilon_1 = \epsilon_L - \frac{\omega_p^2}{\omega^2} \quad (11)$$

Where 'L' is the back-ground dielectric function of the polar semiconductor and given by [16]

$$\epsilon_L = \frac{\epsilon_\infty\omega^2 - \epsilon_0\omega_t^2}{\omega^2 - \omega_t^2} \quad (12)$$

Equation (11) with the help of equation (12) gives

$$\epsilon_1 = \frac{\epsilon_\infty Y - \epsilon_0}{Y - 1} - \bar{\epsilon} \frac{Z}{T} \quad (13)$$

$$Y = \frac{\omega^2}{\omega_t^2} \quad \& \quad Z = \frac{\omega_p^2}{\omega_t^2}$$

Where the value of ' $\epsilon_1$ ' from equation (13) is substituted in the dispersion relation (10) and vacuum is taken as the binding medium we get the following equation

$$Y[Y^2 - (aZ + b)Y + aZ] = 0 \quad (14)$$

$$a = \frac{B - 2\bar{\epsilon}A}{C - \epsilon_\infty A}, \quad b = \frac{C - \epsilon_0 A}{C - \epsilon_\infty A}$$

The above equation is quadratic in Y thus for each mode 'l' it gives two coupled modes for the given values of the constants.

For the uncoupled SOP mode,  $\omega_p = 0$  or  $Z = 0$  then equation (14) gives,

$$Y_{SOP} = \left( \frac{\omega_{SOP}}{\omega_t} \right)^2 = \left( \frac{C - \epsilon_0 A}{C - \epsilon_\infty A} \right) Z \quad (15)$$

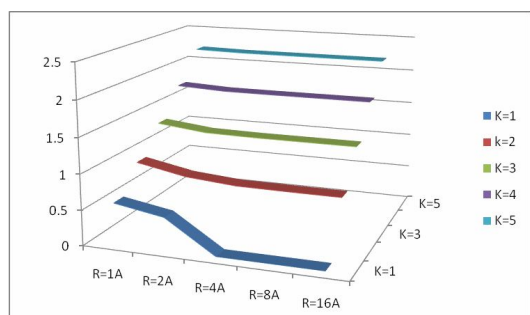
the uncoupled SP mode,  $\epsilon_1 = \bar{\epsilon}$  the dispersion relation (10) then gives,

$$Y_{SP} = \left( \frac{\omega_{SP}}{\omega_t} \right)^2 = \left( \frac{B - 2\bar{\epsilon}A}{C - \bar{\epsilon}A} \right) Z \quad (16)$$

It is evident that the frequency of the uncoupled SP mode depends on the value of  $Z = (\omega_{SP}/\omega_t)^2$  i.e. on the electronic concentration of the conducting materials RbF. The Table for RbF Tubes of different radius is given as

**Table - 1 :** RbF Tubes of Different Radius

$K \backslash \omega_{sp} / \omega_t$	R=1A	R=2A	R=4A	R=8A	R=16A
1	0.576112	0.458197	0	0	0
2	0.949571	0.814658	0.758192	0.753698	0.753417
3	1.34748	1.268568	1.257262	1.256526	1.25648
4	1.768659	1.733666	1.730612	1.73042	1.730408
5	2.210058	2.195285	2.194229	2.194164	2.19416



**Fig.1:** Graph Between Radius of RbF and  $\omega_{sp}/\omega_t$  for Different Incident Wave Vectors in Dielectric Medium.

In order to study the spatial dispersion, as an example we have taken the RbF polar semiconductor figure (1) we observe that the spatial effects depend on the values of wave vector  $\bar{K}$ . The curve for  $\bar{K} = 0$  shows that the spatial dispersion effect are significant for the tube of very small radii ( $R < 16\text{\AA}$ ). If we compare the curve for  $\bar{K} = 5$  with that for  $\bar{K} = 0$ , we observe that the slope of the previous curve decreases more rapidly as compared to the later one. This shows that as we increase the radius, the spatial effects for the larger wave vector values tend to disappear more rapidly as compared to smaller.

#### 4. CONCLUSION

It is clear that the frequencies of Surface Plasmon increases as propagation constant of incident em wave falls on the RbF nanotube because different frequencies of electromagnetic wave have different propagation constant. Frequencies of Surface Plasmon also depend upon the radius of nanotubes only for long rang wave length. Thus RbF nanotube also have radiative properties because this substance only that frequencies which absorbed by its surface Plasmon from external sources. Surface Plasmon frequencies become constant for shorter wavelenghs of incident i.e. for large frequencies of incident electromagnetic waves.

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